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Non-linear thermomagnetic effects due to the arbitrary drag of electrons and phonons near the acoustic instability threshold

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Abstract. The thermopower and the transverse Nernst–Ettingshausen (NE) effect are studied in impurity semiconductors and semimetals exposed to external electric E and non-quantising magnetic H fields in the presence of a lattice temperature gradient with due regard of the non-diffusion approximation for the acoustic phonons. In the absence of electron and phonon heating, the thermal and mutual drags are taken into account. It is shown that in the non-diffusion approximation, in contrast to the diffusion one, the thermal and mutual drags lead to a non-linear current E dependence without electrons and phonons being heated ('non-heated non-linearity'). This leads to the E and H dependence of the thermopower and NE coefficient.

1. Introduction

In an earlier publication [1] it has been shown that in piezosemiconductors, placed in an external electric field, ultrasound introduced externally into the crystal is amplied by the drift of current carriers. Thermal noise, i.e. the intrinsic phonons of the crystal, is also amplified along with the externally introduced phonons. Then, in another study [2] amplification of ultrasound has been obtained in Ge, i.e. in a crystal with deformation-type interaction, whereas in [3] a saturation of current in piezoelectric semiconductors has been observed.

Esaki [4] obtained generation of phonons in bismuth in crossed electric $\boldsymbol{E}$ and quantising magnetic $\boldsymbol{H}$ fields. He has shown that the current–voltage characteristic (CVC) of Bi has an upward break at a certain critical electric field E_{cr}, which corresponds to the electron drift velocity v, equal to the average sound velocity s in the crystal. At $E = E_{cr}$ the current grows sharply. As H grows, E_{cr} increases too.

Thus, amplification was imparted to an externally introduced sound flux and intrinsic phonons in semiconductors with both deformation and piezoelectric interactions between the electrons and phonons, as well as in the semimetal Bi in external electric and magnetic fields. It has been established that at $H = 0$ the CVC has a downward break (it is being saturated) [3], whereas in a magnetic field crossed with an electric one it exhibits an upward break [4].

Later, CVC of the type obtained in [3] for Bi were observed in InSb [5] as well as in $Bi_{1-x}Sb_x$ semiconductive alloys [6].

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Zylbersztejn [7] obtained generation of a high frequency in n-Ge and then also in n-InSb in a strong electric field.

Thus, acoustic instability of semiconductors and semimetals was discovered in an external electric field. Shortly after, the phenomenological theory of acoustoelectronic phenomena was developed. The current saturation in piezoelectrics was explained by the appearance of an acoustoelectric current. Translated into the 'microscopic (kinetic) language', the acoustoelectric current is the drag current.

Inasmuch as the lifetime of acoustic phonons decreases rapidly with temperature, their substantial non-equilibrium conditions are satisfied at low temperatures or under acoustic instability conditions at any temperatures. For acoustic phonons the frequency values of the phonon–electron relaxation are much less than those for optical phonons, and that is why for the noticeable influence of the non-equilibrium on the transport effects it is necessary to wait for microseconds [8, 9]. Owing to the development of laser technology, in the last few years interest has grown again into the effects of phonon generation and amplification. In experiments involving excitation and probing by means of picosecond laser pulses intense amplification of the polar optical phonons was observed [8, 9]. GaAs heterolayers with a two-dimensional electron gas exhibit particularly favourable conditions in this respect [10, 11].

A consistent microscopic theory of CVC for semiconductors and semimetals in external electric and magnetic fields with due regard for possible generation of phonons by the drift of current carriers must be based on the solution of a bound system of kinetic equations for current carriers and phonons, taking into account both the heating of current carriers and phonons and their arbitrary (thermal and mutual) drags. In this case the Boltzmann equation for the phonons should be solved in a non-diffusion approximation. Such a problem was formulated and solved in [12] in a non-diffusion approximation for acoustic phonons and in [13] for optical phonons, too. The same reports have shown that the non-equilibrium of phonons due to their heating and generation alters the transport effects considerably, leading in particular to current saturation at $H = 0$ and to a CVC break with $\boldsymbol{E} \perp \boldsymbol{H}$ fields in accordance with the results of experimental studies [3, 4].

Kocevar [8] has considered the optical phonon non-equilibrium (generation) using a procedure similar to that employed in [12], and has made a number of simplifying assumptions:

(i) In an alternating current field and under laser excitation a spatially uniform case of transfer was considered, in spite of the importance of taking into account the non-uniformity.

(ii) The electron distribution function was considered to be shifted to the drift velocity (Maxwell function). Too high concentrations of current carriers are necessary to realise the assumption (ii).

Proceeding to the statement of the problem posed, it should be noted that the traditional approximation of a small anisotropy of the phonon distribution function $\mathcal{N}_a(q) \ll \mathcal{N}_s(q)$, used in reports on this subject (the so-called 'diffusion approximation'), is applicable to phonons at drift velocities u much less than the average sound velocity s in the crystal (see [12]). In the presence of the external electric and magnetic fields this condition is obviously not fulfilled. This violation shows up in a particularly severe way under acoustic instability conditions, when phonon generation or amplification by charge carriers starts [1, 7]. Actually, as follows from the results of [12], both the symmetrical $\mathcal{N}_s(q)$ and antisymmetrical $\mathcal{N}_a(q)$ parts of the phonon distribution functions, as well

as the ratio $\mathcal{N}_a(q)/\mathcal{N}_s(q)$, increase with u; moreover $\mathcal{N}_a/\mathcal{N}_s \to 1$ when $u \to s$ and $(\mathcal{N}_a/\mathcal{N}_s) \gg 1$ when $u \gg s$. Besides, the solution of the Boltzmann equation for phonons $\mathcal{N}(q)$ is stationary for $u < s$ and non-stationary for $u > s$. These results were obtained in [12] by solving the non-stationary transport equation for phonons without using a diffusion approximation in the absence of spatial non-uniformities (see Appendix).

The present report deals with a theoretical investigation of thermopower and the transverse Nernst–Ettingshausen (NE) effect in impurity semiconductors and semimetals exposed to external electric and non-quantised magnetic fields when there is a temperature gradient ∇T (T is the lattice temperature in terms of the energy units). The investigation is carried out in a non-diffusion approximation for acoustic phonons with due regard to the electron drag by phonons (the thermodrag) and their mutual drag. A similar problem was solved for both metals [14] and semiconductors [15, 16] in a diffusion approximation.

In order to reveal and correlate the net relative contributions of the effects of thermal and mutual drags to the thermomagnetic coefficients near the acoustic instability threshold, we do not consider here the possible heating of electrons and phonons by the electric field.

In the case of an arbitrary drag of phonons and electrons the phonon part of the NE coefficient Q_p differs from zero only for the non-parabolic spectrum of electrons [17, 18]. That is why we consider the case of the Kane spectrum in the two-band approximation for the degenerate electron statistics

$$p(\varepsilon) = (2m_n)^{1/2}(\varepsilon + \varepsilon^2/\varepsilon_g)^{1/2} \tag{1}$$

and the simple energy–momentum dependence for the non-degenerate one

$$p(\varepsilon) = \mu\varepsilon^{s_0}. \tag{2}$$

Here m_n is the electron effective mass at the bottom of the conduction band, ε_g is the band gap, $\mu = (2m_n)^{1/2}$, $s_0 = \frac{1}{2}$ for the parabolic and $\mu = (2m_n/\varepsilon_g)^{1/2}$, $s_0 = 1$ for the strongly non-quadratic spectra.

In this paper general expressions for thermopower and NE coefficient are given for both the degenerate and the non-degenerate statistics of current carriers. In the case under consideration, when there is no heating of electrons and phonons, the dependences of thermopower and the NE coefficient on E and H are due to thermal and mutual drags, i.e. to the dependence of the phonon drift velocity u on E and H. Since this dependence is the same for both non-degenerate and degenerate statistics, in order to shorten our paper we give the concrete dependences of the thermopower and NE coefficient upon E and H for degenerate statistics only.

It is shown that the electron drag by phonons and their mutual drag in the absence of electron and phonon heating lead to a non-linear dependence of current on the E field ('non-heated non-linearity') and hence a dependence of thermopower and the NE coefficient of E and H.

It should be noted, for comparison, that in a diffusion approximation the account of drag leads to the change of the dependences of thermopower and NE coefficient on the lattice temperature only.

2. Fundamental equations and their solutions

Since the mutual electron–phonon drag leads to a decrease of the electron drift velocity $v(\varepsilon)$, the condition $v \ll v_0$ for electrons is satisfied for all values of fields and temperatures

used, where $v_0 = (2T/m)^{1/2}$ for the non-degenerate and $v_0 = v_F$ for the degenerate statistics, and v_F is the Fermi velocity of electrons. Therefore for the electrons a diffusion approximation may be used:

$$f(\varepsilon) = f_s(\varepsilon) + f_a(\varepsilon) \qquad |f_a(\varepsilon)| \ll f_s(\varepsilon). \tag{3}$$

Here $f_s(\varepsilon)$ is the equilibrium Fermi distribution function with temperature T, and $f_a(\varepsilon)$ is the antisymmetric part of the electron distribution function. In order to preserve the usual pattern of calculating the thermomagnetic effects, we divide the phonon distribution function $\mathcal{N}(q)$ into symmetric $\mathcal{N}_s(q)$ and antisymmetric $\mathcal{N}_a(q)$ parts:

$$\mathcal{N}(q) = \mathcal{N}_s(q) + \mathcal{N}_a(q). \tag{4}$$

It should be noted that no restrictions are imposed on the relative quantities $\mathcal{N}_s(q)$ and $\mathcal{N}_a(q)$.

From Boltzmann transport equations for the electrons and phonons by using (1) and (4) we obtain

$$\begin{aligned}
&\boldsymbol{v}(\varepsilon) - \frac{\omega_H(\varepsilon)}{\nu(\varepsilon,u)}[\boldsymbol{h}\boldsymbol{v}(\varepsilon)] + \frac{1}{m(\varepsilon)\nu(\varepsilon,u)}\left\{e\boldsymbol{E}' + \left(\frac{\varepsilon-\zeta_0(T)}{T}\right)\boldsymbol{\nabla}T\right. \\
&\qquad + \frac{4\pi m^2(\varepsilon)}{(2\pi\hbar)^3 p^3(\varepsilon)}\int_0^{2p} W_q q^2\,\mathrm{d}q\,\delta(u) \\
&\qquad \left.\times\left[\frac{s}{\beta(q)}\hbar\omega_q\boldsymbol{\nabla}\mathcal{N}(q,T) - q\boldsymbol{u}\mathcal{N}(q,T)\right]\right\} = 0 \qquad (5)\\
&\mathcal{N}_a(q) + \frac{s}{\beta(q)}\left(\frac{\boldsymbol{q}\boldsymbol{\nabla}\mathcal{N}_s(q)}{q}\right) = \mathcal{N}_s(q)\left(\frac{\boldsymbol{u}\boldsymbol{q}}{\hbar\omega_q}\right) \\
&\mathcal{N}_s(q) + \frac{s}{\beta(q)}\left(\frac{\boldsymbol{q}\boldsymbol{\nabla}\mathcal{N}_a(q)}{q}\right) = \mathcal{N}(q,T) + \mathcal{N}_a(q)\frac{(\boldsymbol{u}\boldsymbol{q})}{\hbar\omega_q} + \frac{\beta_d}{\beta}\mathcal{N}_0(q)
\end{aligned}$$

where

$$\mathcal{N}_0(q) = \frac{1}{4\pi}\int \mathcal{N}(q)\,\mathrm{d}\Omega_0$$

is the isotropic part of the phonon distribution function,

$$\omega_H(\varepsilon) = \frac{|e|H}{m(\varepsilon)c} \qquad \boldsymbol{h} = \frac{\boldsymbol{H}}{H} \qquad \boldsymbol{E}' = \boldsymbol{E} + \boldsymbol{E}_T + \frac{1}{e}\boldsymbol{\nabla}\zeta_0(T)$$

$\boldsymbol{E}_T$ is the thermomagnetic field, $\zeta_0(T)$ is the chemical potential of electrons, β_d is frequency of phonon collision with the defects, the role of which is mainly played by the crystal boundaries, $\beta(q)$ and $\nu(\varepsilon,u) = \nu_p(\varepsilon,u) + \nu_i(\varepsilon)$ are the total phonon and electron momentum scattering rates due to some scattering centres, respectively, $\mathrm{d}\Omega_0 = 2\pi\sin\gamma\,\mathrm{d}\gamma$, $\gamma = (\boldsymbol{q},\hat{\boldsymbol{u}})$, $\boldsymbol{q}$ is the acoustic phonon momentum; the indices 'p' and 'i' mean the phonons and ionised impurities. Moreover according to [12]

$$\begin{aligned}
&\nu_p(\varepsilon,u) = \frac{4\pi}{(2\pi\hbar)^3 p^2(\varepsilon)}\left(\frac{\mathrm{d}p}{\mathrm{d}\varepsilon}\right)\int_0^{2p} W_q q^3\,\mathrm{d}q\,\mathcal{N}(q,T)\delta(u) \\
&\delta(u) = 3\frac{s^2}{u^2}\left(\frac{s}{2u}\varphi(u) - 1\right) \qquad \varphi(u) = \ln\left|\frac{1+u/s}{1-u/s}\right|.
\end{aligned} \tag{6}$$

The deformation (DA) and piezoelectric (PA) interactions of electrons with acoustic

phonons are considered. In this case $W_q = W_0 q^t$ with $t = +1$ for the DA and $t = -1$ for the PA interactions. The solutions of the phonon equations have the form

$$\mathcal{N}_s(q, u, T) = \mathcal{N}(q, T)\left(1 - \frac{u^2}{s^2}\cos^2\gamma\right)^{-1}$$

$$\mathcal{N}_a(q, u, T) = \mathcal{N}_s(q)\frac{u}{s}\cos\gamma - \frac{s}{\beta(q)}\left(\frac{q\nabla\mathcal{N}_s(q, u, T)}{q}\right) \tag{7}$$

$$\mathcal{N}_0(q, u, T) = \mathcal{N}(q, T)\frac{s}{2u}\varphi(u)$$

where $\mathcal{N}(q, T)$ is the Planck equilibrium function with temperature T.

Having determined the current density $\boldsymbol{j}$ with the aid of (5) and (7) and using the conditions $j_{T_x} = 0$, $j_{T_z} = 0$, and $\nabla_x T = 0$ (the latter condition corresponds to the isothermality in the x direction) for the geometry $\boldsymbol{E} \parallel \boldsymbol{H} \parallel \overline{Oy} \perp \nabla T \parallel \overline{Oz}$ one can find the NE field E_{T_x}. As for the thermoelectric field E_{T_z}, which is determined from the condition $j_{T_z} = 0$, we shall investigate it only for the fields and temperature gradient geometries $\boldsymbol{E} \parallel \boldsymbol{H} \parallel \nabla T \parallel \overline{Oz}$ and $\boldsymbol{E} \parallel \overline{Oy} \perp \boldsymbol{H} \parallel \nabla T \parallel \overline{Oz}$.

Let us present the frequencies of collisions of electrons $\nu(\varepsilon)$ and phonons $\beta(q)$ with scattering centres in the form:

$$\nu(\varepsilon) = \nu(\bar{\varepsilon})\left(1 + 2\frac{\varepsilon}{\varepsilon_g}\right)\left(1 + \frac{\varepsilon}{\varepsilon_g}\right)^{-r}\left(\frac{\varepsilon}{\bar{\varepsilon}}\right)^{-r} \qquad \beta(q) = \beta(T)\left(\frac{qs}{T}\right)^k\left(1 + 2\frac{\bar{\varepsilon}}{\varepsilon_g}\right)^l. \tag{8}$$

Here r is the electron momentum scattering parameter, and $k = 0, 1, t$ when the long-wavelength (LW) phonons are scattered by the crystal boundaries, by the reservoir phonons and by the electrons, respectively. For the phonons being scattered by the electrons ($k = t$) $l = 2$, while in other cases $l = 0$; $\bar{\varepsilon} = T$ for non-degenerate and $\bar{\varepsilon} = \zeta$ for degenerate electrons; ζ is the Fermi energy.

Since the electron part of the thermopower remains unchanged, we shall further study the phonon part of the integral thermopower V_p as well as the electronic Q_e and phonon Q_p parts of the NE coefficient.

3. Thermopower

If the phonons are mainly scattered by the electrons ($k = t$), then in a zero approximation on the degeneracy the phonon part of the thermopower has the form

$$V_p = V_{p0}\delta(u) = 8\frac{\Delta T}{e}\frac{\beta_e(T)}{\beta(T)}\frac{s^2}{u^2}\left(\frac{s}{2u}\varphi(u) - 1\right) \tag{9}$$

where $\Delta T = T(+0) - T(L_z)$, L_z being the linear dimension of the specimen in the z direction.

From (9) it is seen that approaching the acoustic instability threshold ($u \to s$) V_p is sharply increased because $\varphi(u) \to \infty$ when $u \to s$.

3.1. The case $\boldsymbol{E} \parallel \boldsymbol{H} \parallel \nabla T \parallel \overline{Oz}$

If the electrons are scattered by the impurity ions ($\nu_i(\varepsilon) \gg \nu_p(\varepsilon, u)$), then

$$u = s\left(\frac{E}{E_i}\right) \qquad E_i = \left(\frac{m(\zeta)\nu_i(\zeta)s}{e}\right)\frac{\beta}{\beta_e}. \tag{10}$$

As follows from this expression, $u = s$ when $E = E_i$ and $u \ll s$ when $E \ll E_i$. In the fields $E \ll E_i$, expanding $\varphi(u)$ is a power series of (u/s), from (9) we have

$$V_{pi} = V_{p0}[1 + \tfrac{3}{5}(E/E_i)^2]. \tag{11}$$

It is seen that V_{pi} increases with increasing E.

In the region of moderate values of the phonon drift velocity when $0 < u < s/2$ (the range of fields $0 < E < E_i$ satisfy this condition) with an accuracy to the logarithmic dependence on E the function $\varphi(u) = \text{const}$ and we obtain

$$V_{pi} = 3V_{p0}\left(\frac{E_i}{E}\right)^2\left(\frac{E_i}{2E}\varphi(u) - 1\right) \simeq \tfrac{3}{2}V_{p0}\left(\frac{E_i}{E}\right)^3\varphi\left(\frac{E}{E_i}\right). \tag{12}$$

When $u \to s$

$$V_{pi} = \tfrac{3}{2}V_{p0}\varphi(u). \tag{13}$$

From (12) and (13) it is seen that in the range of moderate phonon drift velocities the thermopower decreases with increasing electric field as E^{-3}, whereas just at the acoustic instability point it grow sharply at the expense of $\varphi(u)$.

If the phonons and electrons are mainly scattered by each other then in the immediate vicinity of the acoustic instability point ($u \to s$) we obtain

$$u = s\frac{\exp(E/E_p) - 1}{\exp(E/E_p) + 1} \qquad \varphi(u) = \frac{E}{E_p} \qquad E_p = \frac{3}{2}\frac{\beta_{pb}}{\beta}\frac{m\nu_p s}{e} \tag{14}$$

where $\beta_{pb} = \beta_p + \beta_b$, β_p and β_b being the collision frequencies of LW phonons by the reservoir phonons and by the crystal boundaries on energy or momentum transfer, respectively. As follows from (14) in fields $E \gg E_p$ the quantity $u = s = \text{const}$. Then

$$V_p = \tfrac{3}{2}V_{p0}\varphi(u) = \tfrac{3}{2}V_{p0}(E/E_p) \gg V_{p0}. \tag{15}$$

In other words, V_p grows with increasing electric field as E. From (14) it is also seen that when $E = E_p$ the phonon drift velocity is

$$u = 0.46s \simeq s/2. \tag{16}$$

This shows that the range of moderate phonon drift velocities has in fact an upper limit $\simeq s/2$. When $0 < u < s/2$ we find in a way similar to (12) that

$$u \simeq s(E_p/E)^{1/2}\varphi^{1/2}(u) \qquad V_p \simeq \tfrac{1}{2}(E/E_p)^{3/2}\varphi^{-1/2})(E_p/E) \tag{17}$$

i.e. in contrast to (12) in the case of mutual drag in the range of moderate phonon drift velocities with an accuracy of the factor $\varphi(u)$ the thermopower V_p grows with the field as $E^{3/2}$.

In the case of $E \ll E_p$ we find from (9) that

$$V_p = V_{p0}[1 + \tfrac{3}{5}(E_p/E)^2]. \tag{18}$$

Expressions (11), (14), (15) and (17) show that both the electron drag by phonons and

their mutual drag in the absence of electron heating lead to a non-linear dependence of the current on the electric field (non-heated non-linearity) and as a consequence to the increase of thermopower with the field E.

3.2. *The case of* $\boldsymbol{E} \parallel \overline{Oy} \perp \boldsymbol{H} \parallel \nabla T \parallel \overline{Oz}$

In a strong magnetic field in a zero approximation on scattering we have

$$u = \frac{\beta_e}{\beta}\frac{cE}{H} = s\left(\frac{E}{E_k}\right)$$

(independent of the scattering mechanism). Then according to (9), V_p is of the form

$$V_p = 3V_{p0}\left(\frac{E_k}{E}\right)^2\left(\frac{E_k}{2E}\varphi(u) - 1\right). \tag{19}$$

As seen from this expression, when $u \ll s$ we have

$$V_p = V_{p0}[1 + \tfrac{3}{5}(E/E_k)^2]. \tag{20}$$

In the range of moderate values of the drift velocity $0 < u < s/2$, i.e. when $E < E_k$, we have

$$V_p \sim (E_k/E)^3\varphi(E/E_k) \tag{21}$$

i.e. with an accuracy of a logarithmic dependence, the thermopower decreases with increasing E and grows because of increasing $\varphi(u)$ when $u \to s$.

With $E \to E_k$, i.e. when $u \to s$, the thermopower is determined by the expression

$$V_p = \tfrac{3}{2}V_{p0}\varphi(E/H) \tag{22}$$

and it is sharply increased because of growing $\varphi(E/H)$.

We also present the expression of V_p for non-degenerate electrons:

$$V_p = \frac{\Delta T}{e}\frac{8}{3}\frac{s_0^2\Gamma(1 + s_0(3 + 2r))}{\Gamma(3 + s_0(2r - 1))}\frac{\beta_e^{(0)}(T)\beta_e(T)}{\beta^2(T)}\Omega^2\delta(u) \tag{9a}$$

where

$$\Omega = \frac{\mu^2 T^{2s_0 - 1}}{m_n} \qquad \beta_e^{(0)}(T) = \frac{4\pi s W_0 m_n^2}{(2\pi\hbar)^3}\left(\frac{T}{s}\right)^t$$

and $\Gamma(\psi)$ is Euler's gamma function.

At $\boldsymbol{E} \parallel \boldsymbol{H} \parallel \nabla T$ in the case of scattering of the electrons and phonons mainly by each other we get

$$E_p = \frac{3s_0^2}{2^{1+t/2}}\frac{\Gamma(4s_0 - 1)}{\Gamma(1 - s_0 t)}\frac{m_n\nu_p(T)s}{e}\frac{\beta_{pb}(T)}{\beta_e(T)}\Omega^{2+t/2} \tag{14a}$$

whereas in the case of thermal drag we have

$$E_i = \frac{2^{3/2}s_0^2\Gamma(4s_0 - 1)}{\Gamma(3s_0 + 1)}\frac{\beta(T)}{\beta_e(T)}\frac{m_n v_i(T)s}{e}\Omega^{1/2}. \tag{10a}$$

4. Transverse Nernst–Ettingshausen effect

If $\overline{Oy} \| \boldsymbol{E} \| \boldsymbol{H} \perp \nabla T \| \overline{Oz}$, a transverse NE field E_{T_x} appears in addition to the thermal field E_{T_z}:

$$E_{T_x} = H(Q_e + Q_p)\nabla_z T. \tag{23}$$

In weak magnetic fields when $\omega_H \ll \nu(\zeta, u)$, in a second approximation on the degeneracy we find

$$\begin{aligned} Q_e &= \frac{1}{eH}\frac{\pi^2}{3}\frac{\omega_H}{\nu(\zeta,u)}\frac{(1+\alpha)^{r-1}}{(1+2\alpha)}\left(\frac{I}{\zeta}\right)A(\alpha,r) \\ Q_p &= \frac{1}{eH}\frac{32\pi^2}{9}\frac{\beta_e(T)}{\beta(T)}\frac{\omega_H}{\nu(\zeta,u)}\frac{\alpha(1+\alpha)^{r-1}}{(1+2\alpha)^2}\left(\frac{T}{\zeta}\right)^2 A(\alpha,r)\delta(u) \\ A(\alpha,r) &= \left(r - \frac{4\alpha(1+\alpha)}{(1+2\alpha)^2}\right) \end{aligned} \tag{24}$$

whereas in strong magnetic fields ($\omega_H \gg \nu(\zeta, u)$) we have

$$\begin{aligned} Q_e &= \frac{1}{eH}\frac{\pi^2}{3}\frac{1}{(1+\Lambda)^l}\frac{\nu(\zeta,u)}{\omega_H}\frac{(1+2\alpha)^3}{(1+\alpha)^{r+1}}\left(\frac{T}{\zeta}\right)A(\alpha,r) \\ Q_p &= \frac{1}{eH}\frac{32\pi^2}{9}\frac{\nu(\zeta,u)}{\omega_H}\frac{\alpha(1+2\alpha)^2}{(1+\alpha)^{r+1}}\frac{\delta(u)}{(1+\Lambda)^l}\frac{\beta_e(T)}{\beta(T)}\left(\frac{T}{\zeta}\right)^2 A(\alpha,r) \end{aligned} \tag{25}$$

where $\alpha = \zeta/\varepsilon_g$

$$\Lambda = \frac{\beta(\alpha)}{\beta_{pb}(\alpha)}(1+2\alpha)^2(1+\alpha)^{t/2} \gg 1$$

because of $\beta/\beta_{pb} \gg 1$, whereas $\nu(\zeta)$ and ω_H are the values of the respective quantities in the parabolic case.

From (24) and (25) it follows that when the electrons are scattered by the impurity ions ($\nu_i(s) \gg \nu_p(\zeta, u)$), Q_e does not depend on E, while the phonon part Q_p depends on E as $\delta(E/E_i)$. But if the electrons are scattered by phonons, in weak magnetic fields $Q_e \sim (\delta(E/E_p))^{-1}$, whereas Q_p is independent of E and H (i.e. Q_p tends to a constant value). In contrast, in strong magnetic fields in the case of a mutual drag $Q_e \sim \delta(u/s)$, whereas $Q_p \sim \delta^2(u/s)$. From (24) and (25) it is also seen that in the case of electron scattering by piezoacoustic phonons ($r = \frac{1}{2}$) both Q_e and Q_p reverse their signs with growing non-parabolicity parameters α. The critical value of α at which Q_e and Q_p become equal to zero is $\alpha_{cr} = 0.215$.

Now we present the dependences of Q_e and Q_p on E in specific cases (the dependences of these quantities on H remain the same as they are in the linear theory).

4.1. The case of electron scattering by impurity ions (thermal drag)

In this case the dependences of Q_p on E are the same as those of V_p on E, determined by the expressions (11) to (13). Consequently, in the case of the thermal drag Q_p, by analogy with V_p, increases with increasing E in the region of small drift velocity values ($u \ll s$) and near the acoustic instability threshold, and it decreases in the region of moderate phonon drift velocities ($0 < u < s/2$) in both weak and strong magnetic fields.

4.2. The case of mutual drag ($k = t$; $\beta_{pb}/\beta \gg \nu_i(\zeta)/\nu_p(\zeta, u)$)

In a weak magnetic field

$$\begin{aligned} Q_e &= Q_{e0}[1 + \tfrac{3}{5}(E/E_p)^2]^{-1} && \text{if } u \ll s \\ Q_e &= \tfrac{1}{2}Q_{e0}(E_p/E)^{3/2}\varphi^{1/2}(E_p/E) && \text{if } 0 < u < s/2 \\ Q_e &= Q_{e0}\varphi^{-1}(u/s) && \text{if } u \to s \end{aligned} \tag{26}$$

and

$$Q_p = Q_{p0} = \frac{1}{eH}\frac{32\pi^2}{9}\frac{\beta_e(T)}{\beta(T)}\frac{\omega_H}{\nu_p(\zeta)}\frac{\alpha(1+\alpha)^{r-1}}{(1+2\alpha)^2}\left(\frac{T}{\zeta}\right)^2 A(\alpha, r) \tag{26a}$$

for the whole region of values of u.

From (26*a*) it is seen that Q_p tends to saturation, whereas the electron part of the NE coefficient $Q_e \sim E_p/E$ when $u \to s$.

If the magnetic field is strong, then for $u \ll s$

$$\begin{aligned} Q_e &= Q_{ep}(H)\delta(u/s) = Q_{ep}(H)[1 + \tfrac{3}{5}(E/E_p)^2] \\ Q_p &= Q_{pp}(H)\delta^2(u/s) = Q_{pp}(H)[1 + \tfrac{8}{5}(E/E_p)^2] \end{aligned} \tag{27}$$

and for $0 < u < s/2$

$$\begin{aligned} Q_e &= Q_{ep}(H)\left(\frac{E}{E_p\varphi(E)}\right)\left[\frac{1}{2}\left(\frac{E}{E_p}\varphi(u)\right)^{1/2} - 1\right] \sim \left(\frac{E}{E_p}\right)^{3/2}\varphi^{-1/2}\left(\frac{E_p}{E}\right) \\ Q_p &= Q_{pp}(H)\left(\frac{E}{E_p\varphi(u)}\right)^2\left[\frac{1}{2}\left(\frac{E}{E_p}\right)^{1/2}\varphi^{1/2}(u) - 1\right]^2 \sim \left(\frac{E}{E_p}\right)^3\varphi^{-1}\left(\frac{E_p}{E}\right) \end{aligned} \tag{28}$$

and, finally, near the acoustic instability point ($u \to s$)

$$\begin{aligned} Q_e &= Q_{ep}(H)\varphi(u/s) \sim (E/E_p) \\ Q_p &= Q_{pp}(H)\varphi^2(u/s) \sim (E/E_p)^2. \end{aligned} \tag{29}$$

From the results obtained one can see that in the case of electron scattering by phonons in a strong magnetic field both Q_e and Q_p increase with increasing E in all regions of values of phonon drift velocities $0 < u \leqslant s$.

In conclusion we present the final expressions of Q_e and Q_p for non-degenerate semiconductors.

In weak magnetic fields

$$\begin{aligned} Q_e &= \frac{1}{eH}\frac{\omega_H}{\nu(u, T)}\frac{b_1(s_0, r)}{\Omega^{2-r}} \\ Q_p &= \frac{8}{3}\frac{b_2(s_0, r)}{eH}\frac{\beta_e(T)}{\beta(T)}\frac{\omega_H}{\nu(u, T)}\Omega^r\delta(u) \end{aligned} \tag{30}$$

where

$$b_1(s_0, r) = \frac{2^{-r}}{s_0^2}\left(\frac{\Gamma(x(q/2), 6 + s_0(4r-5))}{\Gamma(3 + s_0(2r-1))} - \frac{[3 + s_0(2r-1)]\Gamma(x(q/2), 5 + s_0(4r-5))}{\Gamma(3 + s_0(2r-1))}\right)$$

$$b_2(s_0, r) = \frac{2^{-r}}{s_0^2}\left(1 - \frac{\Gamma(1 + s_0(3 + 2r))\Gamma(x(q/2), 5 + s_0(4r-5))}{\Gamma^2(3 + s_0(2r-1))}\right)$$

whereas in strong magnetic fields

$$Q_e = \frac{2^r s_0^2}{eH}\frac{\nu(u,T)}{\omega_H}\frac{\Gamma(s_0(7-2r)-1)}{\Gamma(3s_0+1)}\left\{2(r-2s_0+1)\right.$$
$$+\left(\frac{\Omega}{2}\right)^{r+t/2}\frac{(\beta_e/\beta)}{\Gamma(s_0(7-2r)-1)}\frac{\nu_p(u,T)}{\nu(u,T)}$$
$$\left.\times\left[\int_0^\infty x^{3s_0+1}\mathscr{D}(x(q/2))\,e^{-x}\,dx-\int_0^\infty x^{3s_0}\mathscr{D}\left(x\left(\frac{q}{2}\right)\right)e^{-x}\,dx\right]\right\} \tag{31}$$

$$Q_p = \frac{1}{eH}\frac{2^{3+r}s_0^4}{3\Gamma(1+3s_0)}\frac{\nu(u,T)}{\omega_H}\frac{\beta_e^{(0)}(T)}{\beta(\varepsilon_g,T)}\Omega\delta(u)$$
$$\times\left\{\frac{\Gamma(7s_0-1)}{\Gamma(3s_0+1)}\Gamma(s_0(7-2r)-1)-\Gamma(s_0(11-2r)-3)\right.$$
$$+\left(\frac{\Omega}{2}\right)^{r+t/2}\frac{\beta_e(\varepsilon_g,T)}{\beta(\varepsilon_g,T)}\frac{\nu_p(u,T)}{\nu(u,T)}\left[\int_0^\infty x^{7s_0-2}\mathscr{D}\left(x\left(\frac{q}{2}\right)\right)e^{-x}\,dx\right.$$
$$\left.\left.-\frac{\Gamma(7s_0-1)}{\Gamma(3s_0+1)}\int_0^\infty x^{3s_0}\mathscr{D}\left(x\left(\frac{q}{2}\right)\right)e^{-x}\,dx\right]\right\} \tag{32}$$

where

$$\mathscr{D}\left(x\left(\frac{q}{2}\right)\right)=\frac{\Gamma(x(q/2),s_0(8+t)-3)}{\Gamma(x(q/2),4s_0-1)}\qquad x(q/2)=\frac{\varepsilon(q/2)}{T}$$

and $\Gamma(x(q/2),\psi)$ is the incomplete gamma function. It should be noted that the terms proportional to $\nu_p(u,T)/\nu(u,T)$ are essential in (31) and (32) only in the case of mutual drag.

The above expressions for Q_p show that in the case of a parabolic spectrum ($\alpha=0$ or $s_0=\frac{1}{2}$) $Q_p=0$ for both degenerate and non-degenerate charge carriers, which is in accordance with the results of other studies [17, 18].

5. Discussion of results

As follows from (11) to (13), in the case of $\boldsymbol{E}\parallel\boldsymbol{H}$ the thermopower caused by thermal drag grows with the electric field in the immediate vicinity of and far from the acoustic instability point and decreases with E in the region of moderate drift velocities. As for thermopower associated with mutual drag, it grows with E over the whole region of phonon drift velocities, as seen from (15) to (18).

In a strong $\boldsymbol{E}\perp\boldsymbol{H}$ magnetic field the thermopower grows near and far from the acoustic instability point, whereas in the region $0<u<s$ it decreases with E and grows with H.

According to (24) the electron part of the NE coefficient is independent of E, whereas Q_p depends on E, if the electrons are scattered by impurity ions. But in the case of

electron scattering by phonons in weak magnetic fields $Q_e \sim \delta^{-1}(E/E_p)$, whereas Q_p is independent of E and H, i.e. it is saturated.

In a strong magnetic field $Q_e \sim \delta(u)$ and $Q_p \sim \delta^2(E/E_p)$, if the mutual drag of electrons and phonons is substantial.

In the case of thermal drag $Q_p \sim \delta(u)$, whereas Q_e is saturated. When the electrons are scattered by piezoacoustic phonons ($r = \frac{1}{2}$), Q_e and Q_p reverse their signs and turn to zero at a certain critical value of the non-parabolicity parameter α, according to (24) and (25).

Let us evaluate the order of magnitude of the thermopower in the cases of thermal and mutual drags near the acoustic instability threshold. From (9) and (13) it is seen that when $u \to s$ ($u \simeq s$)

$$V_p = 4\frac{\Delta T}{e}\frac{\beta_e(T)}{\beta(T)}\varphi(u)$$

and grows sharply with $\varphi(u)$. However, we shall show that $\varphi(u)$ has an upper limit. For this purpose we first assumed that the electrons have been heated, whereas the temperature of LW phonons coincides with the short-wavelength (SW) phonon reservoir temperature, as before. Then we have the following expression for the dimensionless electron temperature $\theta_e = T_e/T$:

$$\theta_e = 1 + \left[\frac{1}{3}\left(\frac{E}{E_i}\right)^2 \frac{\nu_i(\zeta)}{\nu_p(\zeta)} - \frac{s}{2u}\frac{\varphi(u)}{\eta}\right] \qquad \eta = \frac{\beta}{\beta_{pb}}.$$

Now, if one cools the electrons to the lattice temperature, then $\theta_e = 1$. From this condition we find

$$\varphi(u) = \frac{2}{3}\left(\frac{E}{E_i}\right)^3 \frac{\nu_i(\zeta)}{\nu_p(\zeta)}\eta.$$

For InSb with parameters $m_n = 3 \times 10^{-29}$ g, $\nu_i(\zeta) = 10^{11}$–10^{12} s^{-1}, $\nu_p(\zeta) = 10^{10}$–10^{11} s^{-1}, $s = 3 \times 10^5$ cm s^{-1} and $\eta = 10^3$–10^4, the quantity $E_i = 0.6$ V cm^{-1}, if $\nu_i(\zeta) = 10^{11}$ s^{-1}. With $\nu_i(\zeta) = 10^{12}$ s^{-1} the critical field $E_i = 6$ V cm^{-1}. It means that even at relatively small electric fields the phonon drift velocity is comparable with s. Substituting $\varphi(u)$ into (13), we obtain for $\Delta T = 77$ K and $u \to s$ that $V_{pi} = 1.77 \times 10^3$ V, if $\eta = 10^3$, $\nu_i/\nu_p = 10$. Now we evaluate V_p in the case of mutual drag. For InSb with the above parameters ($\nu_p(\zeta) = 10^{11}$ s^{-1}, $\eta = 10^3$), $E_p = 10^{-3}$ V cm^{-1} and even at $E = 1$ V cm^{-1} the quantity $\varphi(u) = 10^3$. Then, according to (15), we have $V_p = \frac{3}{2}V_{p0}(E/E_p) \simeq 26.6$ V.

The NE EMF (or NE voltage) can be evaluated in a similar way: $U_{e,p} = HQ_{e,p}\Delta TL_x/L_z$, where L_x is the linear dimension of the specimen in the x direction. From (24) it is seen that in weak magnetic fields in the case of a thermal drag Q_e tends to saturation, whereas when $u \to s$ the quantity $U_p = 0.6$ V, if $\alpha = 0.2$, $\omega_H = 10^{10}$ s^{-1}, $\nu_i(\zeta) = 10^{11}$ s^{-1}, $L_x \simeq L_z$, $(T/s) = 10^{-1}$ and $\varphi(u) = 10^4$. In strong magnetic fields we find from (25) near the acoustic instability threshold $U_p \simeq 0.3$ V when $\omega_H = 10^{12}$ s^{-1} and $\nu_i(\zeta) = 10^{11}$ s^{-1}.

Thus, it follows from the foregoing that the thermal and mutual drags yield non-linear dependences of thermopower and NE coefficient on the electric field, which is revealed in a substantial growth of the absolute values of these quantities. In other words, the drags and non-heated non-linearity related to E and H (at $\boldsymbol{E} \perp \boldsymbol{H}$) always accompany each other.

The results of this investigation show that by measuring the thermopower and thermomagnetic coefficients near the acoustic instability point (when the electrons and phonons are in a powerfully non-equilibrium state) it is possible to obtain EMF values exceeding by several orders the EMF values of equilibrium thermal sources.

Appendix

We present a short survey of the results and assumptions discussed in [12] and relating to the solution of the non-stationary Boltzmann equation for phonons interacting with electrons in strong electric and non-quantising magnetic fields. Consideration is given to the case of high concentrations of electrons, when the isotropic part of their distribution function is that of the Fermi (or Maxwell) type with an effective temperature T_e. Besides, the case of a thermal reservoir of the SW phonons is discussed where the use of a one-particle (single-mode) relaxation time describing the thermalisation rate of an individual excited phonon and certainly including the effects of losses at the crystal boundaries ($\tau_p^{-1} = \tau_{pp}^{-1} + \tau_{pb}^{-1}$) is well justified. Here $\beta_p = \tau_{pp}^{-1}$ and $\beta_b = \tau_{pb}^{-1}$ are the relaxation frequencies of the LW phonons at the reservoir phonons and at the crystal boundaries, respectively. The existence of a separate-mode relaxation time makes it possible to write the highly non-linear integral of phonon–phonon and phonon–boundary collisions in terms of relaxation frequencies as [8]

$$(\partial \mathcal{N}/\partial t)_p = -\beta_p(q)[\mathcal{N}(q) - \mathcal{N}_q(T_p)] - \beta_b[\mathcal{N}(q) - \mathcal{N}_q(T_L)]. \tag{A1}$$

Here $\mathcal{N}(q)$ is the actual distribution of phonons, and $\mathcal{N}_q(T_p)$ and $\mathcal{N}_q(T_L)$ are the equilibrium Planck distributions with the temperature of heated phonons T_p and that of the lattice T_L. In the presence of the thermal reservoir, $T_p = T_L$.

The collision integral between the phonons and carriers has the form

$$(\partial \mathcal{N}/\partial t)_c = \sum_{p} W_q\{\mathcal{N}(q,t)[F(\boldsymbol{p}+\boldsymbol{q}) - F(\boldsymbol{p})] + F(\boldsymbol{p}+\boldsymbol{q})[1 - F(\boldsymbol{p})]\}\delta(\varepsilon_{p+q} - \varepsilon_p - \hbar\omega_q). \tag{A2}$$

We present the carrier distribution function as

$$F(\boldsymbol{p}) = F_0(\varepsilon_p) + f(\boldsymbol{p}) \qquad \text{where } |f(\boldsymbol{p})| \ll F_0(\varepsilon_p). \tag{A3}$$

Here $F_0(\varepsilon_p)$ is the equilibrium Fermi distribution function with the effective temperature of carriers, whereas

$$f(\boldsymbol{p},t) = f(\varepsilon_p,t)\frac{\boldsymbol{p}}{p} = V(\varepsilon_p,t)p\left(-\frac{\partial F_0(\varepsilon_p)}{\partial \varepsilon_p}\right)\frac{\boldsymbol{p}}{p} \tag{A4}$$

is the asymmetric part of the distribution function of electrons and $V(\varepsilon_p, t)$ is their drift velocity with energies ε_p.

If we substitute (A3) and (A4) in (A2) and take into account the fact that

$$\beta_c \equiv \tau_{pe}^{-1} = -\sum_{p} W_q[F_0(\varepsilon_{p+q}) - F_0(\varepsilon_p)]\delta(\varepsilon_{p+q} - \varepsilon_p - \hbar\omega_q) \tag{A5}$$

by definition, (A2) can be reduced to the form

$$\left(\frac{\partial \mathcal{N}}{\partial t}\right)_c = \beta_c\left[\mathcal{N}_q(T_e) - \mathcal{N}(q,t)\left(1 - \frac{V(t)\boldsymbol{q}}{\hbar\omega_q}\right)\right]. \tag{A6}$$

Here $\mathcal{N}_q(T_e)$ is the equilibrium Planck distribution function with the temperature of

carriers T_e. With the aid of (A1) and (A6) the Boltzmann equation for phonons can be transformed to the following form:

$$\frac{\partial \mathcal{N}(q,t)}{\partial t} = \beta\Big[-\mathcal{N}(q,t)\Big(1-\frac{\boldsymbol{u}(t)\boldsymbol{q}}{\hbar\omega_q}\Big) + \frac{1}{\beta(q)}[\beta_e(q)\mathcal{N}_q(T_e)+\beta_p(q)\mathcal{N}_q(T_p)+\beta_b^{(\varepsilon)}\mathcal{N}_q(T)]\Big]. \tag{A7}$$

As distinct from [12], we ignored the scattering of phonons by defects when obtaining this expression. Having introduced the notation

$$\tilde{\mathcal{N}}(q,T_i) = \frac{1}{\beta(q)}(\beta_e(q)\mathcal{N}_q(T_e)+\beta_p(q)\mathcal{N}_q(T_p)+\beta_b^{(\varepsilon)}\mathcal{N}(q,T)) \tag{A8}$$

($T_i = T_e$, T_p or T), we can write the general solution of equation (A7) for the initial conditions $\mathcal{N}(q,t) = \mathcal{N}(q,0)$ at $t=0$ as

$$\mathcal{N}(q,t) = \Big\{\mathcal{N}(q,0)+\beta\tilde{\mathcal{N}}(q,T_i)\int_0^t \exp\Big[\beta\int_0^\tau\Big(1-\frac{\boldsymbol{u}(q,\tau')\boldsymbol{q}}{\hbar\omega_q}\Big)\,d\tau'\Big]\,d\tau\Big\} \times \exp\Big[-\beta\int_0^t\Big(1-\frac{\boldsymbol{u}(q,\tau)\boldsymbol{q}}{\hbar\omega_q}\Big)\,d\tau\Big]. \tag{A9}$$

Here

$$\boldsymbol{u}(q,t) = \frac{\beta_e(q)}{\beta(q)}\boldsymbol{V}(t)$$

is the average drift velocity of phonons interacting with electrons.

If the external electric field is constant (i.e. $\omega \to 0$), then (A9) can be written as

$$\mathcal{N}(q,t) = \Big(\mathcal{N}(q,0)-\frac{\tilde{\mathcal{N}}(q,T_i)}{(1-(\boldsymbol{uq})/\hbar\omega_q)}\Big) \times \exp\Big[-\beta\Big(1-\frac{(\boldsymbol{uq})}{\hbar\omega_q}\Big)t\Big]+\frac{\tilde{\mathcal{N}}(q,T_i)}{(1-(\boldsymbol{uq})/\hbar\omega_q)}. \tag{A10}$$

According to (A10), for $(\boldsymbol{uq}) < \hbar\omega_q$ the distribution of phonons is stationary. In fact, in this case from (A9) or (A10) we obtain

$$\lim_{t\to\infty}\mathcal{N}(q,t) \equiv \mathcal{N}(q) = \frac{\tilde{\mathcal{N}}(q,T_i)}{(1-(\boldsymbol{uq})/\hbar\omega_q)}. \tag{A11}$$

For $(\boldsymbol{uq}) > \hbar\omega_q$ the function $\mathcal{N}(q,t)$ is increasing in time, with the amplification coefficient

$$\gamma_q = \frac{\beta}{s}\Big(\frac{(\boldsymbol{uq})}{\hbar\omega_q}-1\Big). \tag{A12}$$

If along with the intrinsic phonons of the crystal $\mathcal{N}_q(T)$ the initial phonon distribution includes externally introduced phonons $\mathcal{N}_{ex}(q)$, the latter are also amplified with the

amplification coefficient (A12). In fact, if there is a flux of externally introduced phonons, then equation (A7) can be written as

$$\frac{\partial \mathcal{N}(q,t)}{\partial t} + \boldsymbol{u}(q)\frac{\partial \mathcal{N}(q,t)}{\partial \boldsymbol{r}} = \boldsymbol{\beta}\left[-\mathcal{N}(q,t)\left(1-\frac{(\boldsymbol{uq})}{\hbar\omega_q}\right) + \tilde{\mathcal{N}}(q,T_i)\right]. \tag{A13}$$

By a simple substitution $t' = t$ and $\boldsymbol{r}' = \boldsymbol{r} - \boldsymbol{u}(q)t'$, equation (A13) is reduced to (A7). The solution of (A13) is as follows:

$$\mathcal{N}(q,\boldsymbol{r},t) = \left\{\mathcal{N}(q,\boldsymbol{r}-\boldsymbol{u}(q)t,0) + \beta\tilde{\mathcal{N}}(q,T_i)\right.$$
$$\left.\times\int_0^t \exp\left[\beta\int_0^\tau\left(1-\frac{(\boldsymbol{uq})}{\hbar\omega_q}\right)\mathrm{d}\tau'\right]\mathrm{d}\tau\right\}$$
$$\times\exp\left[-\beta\int_0^t\left(1-\frac{(\boldsymbol{uq})}{\hbar\omega_q}\right)\mathrm{d}\tau\right]. \tag{A14}$$

If the specimen is considered to be spatially uniform, it is natural to suppose that the initial distribution of phonons is also uniform. Then from (A14) it follows that the distribution will remain uniform during subsequent moments of time t, too, i.e.

$$\mathcal{N}(q,\boldsymbol{r}-\boldsymbol{u}(q)t,0) = \mathcal{N}(q,0) \tag{A15}$$

and (A14) coincides with (A9). Solving (A7) directly under the conditions $\partial\mathcal{N}(q,t)/\partial t = 0$ can also give a stationary solution

$$\mathcal{N}(q) = \tilde{\mathcal{N}}(q,T_i)\left(1-\frac{(\boldsymbol{uq})}{\hbar\omega_q}\right)^{-1} \tag{A16}$$

coinciding with (A11).

For further calculations it is more convenient to divide the function $\mathcal{N}(q)$ into a symmetric $\mathcal{N}_s(q)$ and antisymmetric $\mathcal{N}_a(q)$ part. For this purpose we multiply and divide (A16) by $[1 + (\boldsymbol{uq})/\hbar\omega_q]$, which gives

$$\mathcal{N}(q) \equiv \mathcal{N}_s(q) + \mathcal{N}_a(q)$$
$$= \frac{\tilde{\mathcal{N}}(q,T_i)}{[1-(u^2/s^2)\cos^2\gamma]} + \frac{\tilde{\mathcal{N}}(q,T_i)(u/s)\cos\gamma}{[1-(u^2/s^2)\cos^2\gamma]}. \tag{A17}$$

As seen from this formula, a small anisotropy approximation of the phonon distribution function $\mathcal{N}_a(q) \ll \mathcal{N}_s(q)$ (the so-called 'diffusion approximation') is applicable when the phonon drift velocity is much less than the average sound velocity s in the crystal.

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